Killing forms on Quaternion–Kähler manifolds

– Corrigendum –

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The aim of this note is to fill a gap in the proof of Theorem 6.1 in [1] – stating that every Killing p–form ($p \geq 2$) on a compact quaternion–Kähler manifold M^{4m} ($m \geq 2$) is parallel – which was pointed out by Liana David. This gap is due to two wrong coefficients in the formulas at the middle of page 329. The correct equations read

$$pd^+u = d(L^-u), \qquad (p-1)d^cu = d(Ju+3u), \qquad pd^-u = d(\Lambda^+u) - \delta^cu,$$
$$(p-1)\delta^+u = -2d(Cu), \quad (p-1)\delta^cu = -2d(\Lambda^+u) - 2d^-u, \quad (p-3)\delta^-u = -2d(\Lambda u).$$

The remaining part of the proof works verbatim for $p > 3$, but an extra argument is needed for $p = 2$ and $p = 3$.

The case $p = 2$**.** From the 6 equations above one obtains the vanishing of d^+u, δ^-u, and $\delta^+u + d^cu + 3du$, but no longer that of δ^cu and d^-u. Correspondingly, the proof of Lemma 6.3 fails. Fortunately, an ad hoc argument shows that $du = 0$ in this case. Indeed, the third equation of the system (7) shows that du is an eigenform of $2C - J$ for the eigenvalue 9. On the other hand, Lemma 5.1, Lemma 5.2 and the decomposition

$$\Lambda^3(H \otimes E) = H \otimes [\Lambda_0^{2,1}E \oplus E] \oplus \mathrm{Sym}^3 H \otimes [\Lambda_0^3 E \oplus E]$$

show that the eigenvalues of $2C - J$ on the four summands of $\Lambda^3 M$ are respectively 3, $4m + 5$, 15, and $4m + 11$. For $m \geq 2$, none of them equals 9.

The case $p = 3$**.** The proof works well in this case, except that one does not obtain $\delta^-u = 0$ in Lemma 6.2. However, this relation is not needed until the point (b) at the bottom of page 331. In order to rule out that case, one has to replace the argument given there with the fact that for $p = 3$ and $m \geq 2$, the inequality $p \geq 2m + 1$ is impossible.

References

[1] A. Moroianu, U. Semmelmann, *Twistor forms on quaternion-Kähler manifolds,* Ann. Global Anal. Geom. **28**, 319–335 (2005).