

COMPACT HOMOGENEOUS LCK MANIFOLDS ARE VAISMAN

PAUL GAUDUCHON, ANDREI MOROIANU AND LIVIU ORNEA

ABSTRACT. We prove that any compact homogeneous locally conformally Kähler manifold has parallel Lee form.

Theorem 1 below has been claimed by K. Hasegawa and Y. Kamishima in [1] and [2] and a partial result also appeared in [3]. At the time of writing, it is not clear to us that the arguments presented in [1] and [2] are complete. We here present a complete simple proof of this result.

A Hermitian manifold (M, g, J) is called *locally conformally Kähler* — lcK for short — if, in some neighbourhood of any point of M , the Hermitian structure can be made Kähler by some conformal change of the metric. Equivalently, (M, g, J) is lcK if there exists a *closed* real 1-form θ , called the *Lee form* of the Hermitian structure, such that the *Kähler form* $\omega := g(J\cdot, \cdot)$ satisfies:

$$(1) \quad d\omega = \theta \wedge \omega.$$

A lcK Hermitian structure is called *strictly* lcK if the Lee form θ is not identically zero, and *Vaisman* if θ is a non-zero parallel 1-form with respect to the Levi-Civita connection of the metric g . Equivalently, since θ is closed, a strictly lcK manifold is Vaisman if the *Lee vector field* $\xi = \theta^\sharp$ is Killing, i.e. $\mathcal{L}_\xi g = 0$, where \mathcal{L}_X denote the Lie derivative along a vector field X . In general, the Lee vector field ξ and the vector field $J\xi$ (sometimes called the *Reeb vector field* of the lcK structure) are neither Killing, nor holomorphic (meaning that $\mathcal{L}_\xi J = 0$), but we always have $\mathcal{L}_{J\xi}\omega = 0$, since $\mathcal{L}_{J\xi}\omega = -d\theta + \theta(J\xi)\omega + \theta \wedge \theta = 0$.

By a *compact homogeneous lcK manifold* we mean a compact, connected, strictly lcK manifold (M, g, J, ω) , equipped with a transitive and effective left-action of a (compact, connected) Lie group G , which preserves the whole Hermitian structure, i.e. the Riemannian metric g , the (integrable) complex structure J and the 2-form ω . We then have:

Theorem 1. *Any compact homogeneous lcK manifold (M, g, J, ω) is Vaisman.*

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Before starting the proof of Theorem 1, we recall a number of general, well-known facts, concerning a compact homogeneous manifold (M, g) , equipped with a left-action, effective and transitive, of a (connected) compact Lie group G . Without loss of generality, we assume that the stabilizer G_x of any point x of M in G is connected. We denote by \mathfrak{g} the Lie algebra of G and we fix an Ad_G -invariant positive definite inner product, B , on \mathfrak{g} . The induced (infinitesimal) action of \mathfrak{g} is an injective linear map $\sigma : \mathfrak{a} \mapsto \hat{\mathfrak{a}}$ from \mathfrak{g} to the space, $\text{Vect}(M)$, of (smooth) vector fields on M , defined by:

$$(2) \quad \hat{\mathfrak{a}}(x) = \left. \frac{d}{dt} \right|_{t=0} \exp(t \mathfrak{a}) \cdot x,$$

for any \mathfrak{a} in \mathfrak{g} and any x in M , where $\exp : \mathfrak{g} \rightarrow G$ denotes the exponential map and \cdot the action of G on M . Since the G -action on M is a left-action, σ is an *anti-isomorphism* from \mathfrak{g} to $\hat{\mathfrak{g}}$, equipped with the usual bracket of vector fields: $[\hat{\mathfrak{a}}, \hat{\mathfrak{b}}] = -\widehat{[\mathfrak{a}, \mathfrak{b}]}$.

We denote by Inv the space of G -invariant vector fields on M , which is a Lie subalgebra of $\text{Vect}(M)$. Any element Z of Inv commutes with all elements of $\hat{\mathfrak{g}}$. In particular

$$(3) \quad \text{Inv} \cap \hat{\mathfrak{g}} = \hat{\mathfrak{c}},$$

where $\hat{\mathfrak{c}}$ denotes the image in $\hat{\mathfrak{g}}$ of the center, \mathfrak{c} , of \mathfrak{g} .

For any x in M , we denote by σ_x the map $\mathfrak{a} \mapsto \hat{\mathfrak{a}}(x)$, from \mathfrak{g} to the tangent space $T_x M$ of M at x , and by σ_x^* its metric adjoint, from $(T_x M, g_x)$ to (\mathfrak{g}, B) , so that:

$$(4) \quad B(\sigma_x^*(X), \mathfrak{a}) = g_x(X, \sigma_x(\mathfrak{a})),$$

for any X in $T_x M$ and any \mathfrak{a} in \mathfrak{g} . The kernel, \mathfrak{g}_x , of σ_x , is a Lie subalgebra of \mathfrak{g} , namely the Lie algebra of the stabilizer, G_x , of x in G , whereas the image of σ_x^* in \mathfrak{g} is the B -orthogonal complement, \mathfrak{g}_x^\perp , of \mathfrak{g}_x . The Lie algebra \mathfrak{g}_x acts on $T_x M$ by $\mathfrak{a} \cdot X = [\tilde{X}, \hat{\mathfrak{a}}](x) = (D_X^g \hat{\mathfrak{a}})(x)$ for any \mathfrak{a} in \mathfrak{g}_x and any X in $T_x M$, where \tilde{X} here stands for any local vector field around x such that $\tilde{X}(x) = X$ and D^g denotes the Levi-Civita connection of g . We then have

$$(5) \quad \sigma_x[\mathfrak{a}, \mathfrak{b}] = \mathfrak{a} \cdot \sigma_x(\mathfrak{b}), \quad \sigma_x^*(\mathfrak{a} \cdot X) = [\mathfrak{a}, \sigma_x^*(X)],$$

for any \mathfrak{a} in \mathfrak{g}_x , any \mathfrak{b} in \mathfrak{g} and any X in $T_x M$.

For any chosen point x in M , the evaluation map $\text{ev}_x : Z \mapsto \text{ev}_x(Z) := Z(x)$, from Inv to $T_x M$, is injective, and $Z(x)$ belongs to the space, denoted by $(T_x M)^{\mathfrak{g}_x}$, of those X in $T_x M$ such that $\mathfrak{a} \cdot X = 0$ for any \mathfrak{a} in \mathfrak{g}_x ; conversely, any X in $(T_x M)^{\mathfrak{g}_x}$ is equal to $Z(x)$ for a unique Z in Inv . Then, ev_x is a linear *isomorphism* from Inv to $(T_x M)^{\mathfrak{g}_x}$, whose inverse is denoted by ev_x^{-1} . On the other hand, for any X in $(T_x M)^{\mathfrak{g}_x}$, we have $X = \hat{\mathfrak{b}}(x)$ for some \mathfrak{b} in \mathfrak{g} , which is uniquely defined up to the addition of an element of \mathfrak{g}_x and is such that $[\mathfrak{a}, \mathfrak{b}]$ belongs to \mathfrak{g}_x

for any \mathbf{a} in \mathfrak{g}_x , meaning that \mathbf{b} belongs to the normalizer, $N_{\mathfrak{g}}(\mathfrak{g}_x)$, of \mathfrak{g}_x in \mathfrak{g} ; conversely, for any \mathbf{b} in $N_{\mathfrak{g}}(\mathfrak{g}_x)$, $\hat{\mathbf{b}}(x)$ is the value at x of a (unique) element of Inv . For any chosen x in M , we thus get a linear isomorphism

$$(6) \quad \text{Inv} = N_{\mathfrak{g}}(\mathfrak{g}_x)/\mathfrak{g}_x,$$

according to which $\mathbf{b} \bmod \mathfrak{g}_x$ is identified with $\text{ev}_x^{-1}(\sigma_x(\mathbf{b}))$, for any \mathbf{b} in $N_{\mathfrak{g}}(\mathfrak{g}_x)$. This isomorphism is actually a Lie algebra isomorphism.

In general, the rank, $\text{rk } \mathfrak{l}$, of a Lie algebra \mathfrak{l} of compact type (= the Lie algebra of a compact Lie group) can be defined as the dimension of a maximal abelian Lie subalgebra. In particular, $\text{rk } N_{\mathfrak{g}}(\mathfrak{g}_x) \leq \text{rk } \mathfrak{g}$ and it then follows from (6) that

$$(7) \quad \text{rk } \text{Inv} \leq \text{rk } \mathfrak{g} - \text{rk } \mathfrak{g}_x.$$

Proof of Theorem 1. We now assume that the homogeneous Riemannian manifold (M, g) is equipped with a compatible G -invariant lcK Hermitian structure as explained above. In particular, Inv is J -invariant, since J is G -invariant, and contains the Lee vector field ξ and $J\xi$ (notice however that Inv is *not* a priori a complex Lie algebra, as J is not preserved in general by the elements of Inv). Since the G -action on M preserves the Kähler form ω , it preserves $d\omega$ as well, hence also the Lee form θ . Since, moreover, $d\theta = 0$, it follows that $\theta(\hat{\mathbf{a}})$ is *constant* and that $\theta([\hat{\mathbf{a}}, \hat{\mathbf{b}}]) = 0$, for any $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ in $\hat{\mathfrak{g}}$. Alternatively, θ determines an element, $\tilde{\theta}$, of \mathfrak{g}^* , defined by $\tilde{\theta}(\mathbf{a}) := \theta(\hat{\mathbf{a}})$, which vanishes on the derived Lie subalgebra $[\mathfrak{g}, \mathfrak{g}]$. Being of compact type, \mathfrak{g} splits as $\mathfrak{g} = \mathfrak{c} \oplus \mathfrak{s}$, where $\mathfrak{s} = [\mathfrak{g}, \mathfrak{g}]$ is the semi-simple part of \mathfrak{g} . We infer the existence of \mathfrak{t} in \mathfrak{c} such that $\tilde{\theta}(\mathfrak{t}) = 1$, whose image in $\hat{\mathfrak{c}} = \text{Inv} \cap \hat{\mathfrak{g}}$ will be denoted by T , instead of $\hat{\mathfrak{t}}$; we then have $\theta(T) = 1$. Since θ is closed, the kernel, \mathfrak{g}_0 , of $\tilde{\theta}$ in \mathfrak{g} is a Lie subalgebra of \mathfrak{g} and we get the following B -orthogonal decomposition

$$(8) \quad \mathfrak{g} = \langle \mathfrak{t} \rangle \oplus \mathfrak{g}_0,$$

where $\langle \mathfrak{t} \rangle$ denotes the 1-dimensional subspace of \mathfrak{g} generated by \mathfrak{t} . The 1-form ψ defined by

$$(9) \quad \psi := -\iota_T \omega,$$

is G -invariant, in particular $\mathcal{L}_T \psi = 0$, from which we infer, by using (1): $d\psi = \iota_T d\omega = \theta(T)\omega + \theta \wedge \psi$, hence

$$(10) \quad \omega = d\psi - \theta \wedge \psi.$$

Notice that (10) implies that $\theta \wedge \psi \neq 0$ at each point of M , since ω is everywhere non-degenerate, whereas $\iota_T d\psi = 0$; it follows that the G -invariant vector fields $T, J\xi$ are independent at each point of M . Since

$[T, J\xi] = \mathcal{L}_T(J\xi) = 0$, T and $J\xi$ generate a 2-dimensional abelian Lie subalgebra of \mathfrak{Inv} , so that

$$(11) \quad \text{rk } \mathfrak{Inv} \geq 2.$$

From (10), we readily infer that \mathfrak{c} is either 1-dimensional, generated by T , or 2-dimensional, generated by T and by $J\xi$, which is then an element of $\hat{\mathfrak{c}} = \mathfrak{Inv} \cap \hat{\mathfrak{g}}$. Indeed, suppose that \mathfrak{c} is of dimension greater than 1. There then exists $\mathbf{v} \neq 0$ in \mathfrak{c} such that $\tilde{\theta}(\mathbf{v}) = 0$. Again, we denote by V its image in $\hat{\mathfrak{c}} = \mathfrak{Inv} \cap \hat{\mathfrak{g}}$, instead of $\hat{\mathbf{v}}$. Since ψ is G -invariant, we have $\mathcal{L}_V \psi = 0$, so that, by (10), $\iota_V \omega = \psi(V)\theta$. This implies that V is then equal to a (non-zero) multiple of $J\xi$.

Choose any x in M and denote by \mathbf{w}_x the element of \mathfrak{g} defined by

$$(12) \quad \mathbf{w}_x = \sigma_x^*((JT)(x)),$$

so that $B(\mathbf{w}_x, \mathbf{a}) = g_x(JT, \sigma_x(\mathbf{a})) = -\psi(\sigma_x(\mathbf{a})) = -\psi(\hat{\mathbf{a}}(x))$, for any \mathbf{a} in \mathfrak{g} , where ψ is the G -invariant 1-form defined by (9). In particular, \mathbf{w}_x sits in \mathfrak{g}_0 , since $\psi(T) = 0$, and is B -orthogonal to \mathfrak{g}_x . Denote by $C_{\mathfrak{g}_0}(\mathbf{w}_x)$ the commutator of \mathbf{w}_x in \mathfrak{g}_0 and pick any \mathbf{a} in \mathfrak{g}_0 . Then, \mathbf{a} belongs to $C_{\mathfrak{g}_0}(\mathbf{w}_x)$ if and only if $B([\mathbf{w}_x, \mathbf{a}], \mathbf{b}) = 0$ for any \mathbf{b} in \mathfrak{g}_0 . Since $B([\mathbf{w}_x, \mathbf{a}], \mathbf{b}) = B(\mathbf{w}_x, [\mathbf{a}, \mathbf{b}]) = \psi([\hat{\mathbf{a}}, \hat{\mathbf{b}}](x)) = d\psi(\hat{\mathbf{a}}(x), \hat{\mathbf{b}}(x))$, this occurs if and only if $\iota_{\hat{\mathbf{a}}(x)} d\psi = 0$, if and only if $\iota_{\hat{\mathbf{a}}(x)} \omega - \psi(\hat{\mathbf{a}}(x))\theta(x) = 0$ (by using (10) and $\theta(\hat{\mathbf{a}}) = 0$). We eventually get that \mathbf{a} belongs to $C_{\mathfrak{g}_0}(\mathbf{w}_x)$ if and only if $\hat{\mathbf{a}}(x) = -\psi(\hat{\mathbf{a}}(x))(J\xi)(x) = B(\mathbf{w}_x, \mathbf{a})(J\xi)(x)$. We infer that $\sigma_x(\mathbf{w}_x) = B(\mathbf{w}_x, \mathbf{w}_x)J\xi(x)$ and that

$$(13) \quad C_{\mathfrak{g}_0}(\mathbf{w}_x) = \langle \mathbf{w}_x \rangle \oplus \mathfrak{g}_x,$$

where the sum is B -orthogonal. Since \mathbf{w}_x is contained in a maximal abelian Lie subalgebra of \mathfrak{g}_0 , which is contained in $C_{\mathfrak{g}_0}(\mathbf{w}_x)$, we have: $\text{rk } C_{\mathfrak{g}_0}(\mathbf{w}_x) = \text{rk } \mathfrak{g}_0 = \text{rk } \mathfrak{g} - 1$. It then follows from (13) that

$$(14) \quad \text{rk } \mathfrak{g}_x = \text{rk } \mathfrak{g} - 2.$$

By using (7), we infer $\text{rk } \mathfrak{Inv} \leq 2$, hence, by (11):

$$(15) \quad \text{rk } \mathfrak{Inv} = 2.$$

Being a Lie algebra of compact type of rank 2, with non-trivial center (since it contains T), \mathfrak{Inv} is isomorphic either to the abelian Lie algebra $\mathbb{R} \oplus \mathbb{R}$ or to the unitary Lie algebra $\mathfrak{u}_2 = \mathbb{R} \oplus \mathfrak{su}_2$. Notice that the latter case can only occur if $\mathfrak{c} = \langle \mathfrak{t} \rangle$, since $\hat{\mathfrak{c}} \cong \mathfrak{c}$ is contained in \mathfrak{Inv} , whereas the center of \mathfrak{u}_2 is of dimension 1.

Case 1. If $\mathfrak{Inv} \cong \mathbb{R} \oplus \mathbb{R}$, it is generated by T and $J\xi$, which, as already observed, are independent at each point of M . Since JT belongs to \mathfrak{Inv} , we thus have

$$(16) \quad JT = aT + bJ\xi,$$

for some real numbers a, b , with $b \neq 0$. Now, T preserves ω and J , as do any vector field in $\hat{\mathfrak{g}}$, so JT preserves J , since J is integrable. It follows that $J\xi = \frac{1}{b}(JT - aT)$ preserves J as well. We already observed that $J\xi$ preserves ω for *any* lcK structure: $J\xi$ is then a Killing vector field with respect to g . It then follows from (16) that JT is a Killing vector field. Finally, $\xi = \frac{1}{b}(T + aJT)$ is also a Killing vector field, meaning that the lcK structure is Vaisman.

Case 2. It remains to deal with the case when $\mathfrak{c} = \langle \mathfrak{t} \rangle$ is 1-dimensional and $\mathfrak{lnv} \cong \mathfrak{u}_2 = \mathbb{R} \oplus \mathfrak{su}_2$. For convenience, we normalize B so that $B(\mathfrak{t}, \mathfrak{t}) = 1$ and $B(\mathfrak{w}_x, \mathfrak{w}_x) = 1$, where \mathfrak{w}_x has been defined by (12), so that $\sigma_x(\mathfrak{w}_x) = (J\xi)(x)$ (where x is any chosen point in M). Denote by e_0 a generator of the center \mathbb{R} of \mathfrak{u}_2 , and by e_1, e_2, e_3 a triple of generators of \mathfrak{su}_2 , such that $[e_2, e_3] = e_1$, $[e_3, e_1] = e_2$. Via (6), we may identify e_0 with $\mathfrak{t} \bmod \mathfrak{g}_x$ and e_1 , say, with $\mathfrak{w}_x \bmod \mathfrak{g}_x$, in order that the restriction of ω — given by (10) — to $N_{\mathfrak{g}}(\mathfrak{g}_x)/\mathfrak{g}_x$ coincide with the standard form $\omega_0 = e_0 \wedge e_1 + e_2 \wedge e_3$; we here identify e_0, e_1, e_2, e_3 with their B -duals in \mathfrak{g}^* , so that $\theta = e_0$, $\psi(x) = -e_1$ and $(d\psi)(x) = e_2 \wedge e_3$ (by identifying θ with $\tilde{\theta}$ and, similarly, $\psi(x)$ with $\mathfrak{a} \mapsto \psi(\hat{\mathfrak{a}}(x))$ in $(\mathfrak{g}/\mathfrak{g}_x)^*$). It remains to determine the complex structure J of \mathfrak{lnv} in terms of the generators e_0, e_1, e_2, e_3 . Without loss of generality, the generators X, Y of the corresponding 2-dimensional complex space $\Theta_J^{(0,1)}$ of elements of type $(0, 1)$ in $\mathfrak{lnv} \otimes \mathbb{C} = \mathbb{C}e_0 \oplus \mathbb{C}e_1 \oplus \mathbb{C}e_2 \oplus \mathbb{C}e_3$ can be chosen of the form $X = e_0 + \sum_{i=1}^3 a_i e_i$, $Y = \sum_{i=1}^3 b_i e_i$, where a_i, b_i are complex numbers, which must satisfy the following three conditions:

- (i) $\Theta_J^{(0,1)}$ is ω -Lagrangian, i.e. $\omega(X, Y) = 0$, where ω is extended to a \mathbb{C} -bilinear form on $(\mathbb{R} \oplus \mathfrak{su}_2) \otimes \mathbb{C}$;
- (ii) $\Theta_J^{(0,1)}$ is *involutive*, meaning that $[X, Y] = \lambda X + \mu Y$, for some complex numbers λ, μ ;
- (iii) $g := \omega_0(\cdot, J\cdot)$ must be positive definite on $\mathbb{R} \oplus \mathfrak{su}_2$, meaning that $i\omega(Z, \bar{Z}) > 0$, for any Z in $\Theta_J^{(0,1)}$.

It is easily checked that the first condition (i) is expressed by

$$(17) \quad b_1 + a_2 b_3 - a_3 b_2 = 0,$$

whereas the integrability condition (ii) implies $\lambda = 0$ and is then equivalent to the system

$$(18) \quad \begin{aligned} -\mu b_1 - a_3 b_2 + a_2 b_3 &= 0, \\ a_3 b_1 - \mu b_2 - a_1 b_3 &= 0, \\ a_1 b_2 - a_2 b_1 - \mu b_3 &= 0. \end{aligned}$$

If $b_1 \neq 0$, we infer from (17) that $\mu = -1$, so that, the system (18) has a non-trivial solution in b_1, b_2, b_3 if and only if a_1, a_2, a_3 are related by $\sum_{i=1}^3 a_i^2 + 1 = 0$, the solution then being $b_1 = a_1 a_3 - a_2$, $b_2 = a_2 a_3 + a_1$, $b_3 = a_2^2 + 1$. We thus get $Y = a_3 \sum_{i=1}^3 a_i e_i - a_2 e_1 + a_1 e_2 + e_3 =$

$-a_3 e_0 - a_2 e_1 + a_1 e_2 + e_3 \pmod{X}$. It follows that $\Theta_J^{(0,1)}$ meets the first two conditions (i) and (ii), with $b_1 \neq 0$, if and only if it is generated by X, Y of the form $X = e_0 + \sum_{i=1}^3 a_i e_i$, $Y = -a_3 e_0 - a_2 e_1 + a_1 e_2 + e_3$, with $\sum_{i=1}^3 a_i^2 + 1 = 0$. As for the positivity condition (iii), we easily compute $\omega_0(X, \bar{X}) = \bar{a}_1 - a_1 + a_2 \bar{a}_3 - \bar{a}_2 a_3 = -\omega_0(Y, \bar{Y})$, which shows that (iii) is actually *never* satisfied if $b_1 \neq 0$. We thus have $b_1 = 0$, which, by (17), implies $a_2 b_3 - a_3 b_2 = 0$, so that $Y = b_2 e_2 + b_3 e_3$, whereas $X = e_0 + a_1 e_1 \pmod{Y}$; by changing the notation, $\Theta_J^{(0,1)}$ is then generated by $X := e_0 + a_1 e_1$ and $Y := b_2 e_2 + b_3 e_3$. Moreover, since $[X, Y] = a_1 (-b_3 e_2 + b_2 e_3)$, the integrability condition (ii) is satisfied if and only if $b_2 = k b_3$ and $b_3 = -k b_2$, for some complex number k , which must be equal to $\pm i$. If $k = i$, we have $Y = e_2 - i e_3$, hence $i \omega_0(Y, \bar{Y}) = -2$, which is negative. We thus have $k = -i$, hence $Y = e_2 + i e_3$, whereas $i \omega_0(X, \bar{X}) = 2 \Im(a)$, by setting $a_1 = a$, so that $\Im(a) > 0$. The only suitable complex structures J on $\mathbb{R} \oplus \mathfrak{su}_2$ are therefore of the form

$$(19) \quad \begin{aligned} J e_0 &= \frac{\Re(a)}{\Im(a)} e_0 + \frac{|a|^2}{\Im(a)} e_1, & J e_1 &= -\frac{1}{\Im(a)} e_0 - \frac{\Re(a)}{\Im(a)} e_1, \\ J e_2 &= e_3, & J e_3 &= -e_2, \end{aligned}$$

with $\Im(a) > 0$. Since $e_0 = \mathfrak{t} \pmod{\mathfrak{g}_x}$ and $e_1 = \mathfrak{w}_x \pmod{\mathfrak{g}_x}$ represent T and $J\xi$ respectively in Inv , via (6), it follows that (16) is again satisfied in Inv , with $b = \frac{|a|^2}{\Im(a)} > 0$; we then conclude as before that ξ is Killing, i.e. that the lcK structure is Vaisman. \square

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PAUL GAUDUCHON, CMLS, ÉCOLE POLYTECHNIQUE, UMR 7640 DU CNRS, 91128 PALAISEAU, FRANCE

E-mail address: paul.gauduchon@math.polytechnique.fr

ANDREI MOROIANU, UNIVERSITÉ DE VERSAILLES-ST QUENTIN, LABORATOIRE DE MATHÉMATIQUES, UMR 8100 DU CNRS, 45 AVENUE DES ÉTATS-UNIS, 78035 VERSAILLES

E-mail address: andrei.moroianu@math.cnrs.fr

LIVIU ORNEA, UNIV. OF BUCHAREST, FACULTY OF MATHEMATICS, 14 ACADEMIEI STR, 70109 BUCHAREST, ROMANIA, AND INSTITUTE OF MATHEMATICS "SIMION STOILOW" OF THE ROMANIAN ACADEMY, 21, CALEA GRIVITEI STR, 010702-BUCHAREST, ROMANIA.

E-mail address: lornea@fmi.unibuc.ro, Liviu.Ornea@imar.ro